Supplementary

Aberration integral formulation used in this work:

Electrostatic Lens Aberration Integrals

$$L(z) = \frac{1}{32\phi(z)^{\frac{3}{2}}}(\phi''(z)^2 - \phi(z)\phi''''(z))$$

$$M(z) = \frac{1}{8\sqrt{\phi(z)}}\,\phi''(z)$$

$$N(z) = \frac{1}{2}\sqrt{\phi(z)}$$

$$B = \frac{1}{4\sqrt{\phi(z_0)}}\int_{z_o}^{z_i} \frac{L}{4}h^4 + \frac{M}{2}h^2h'^2 + \frac{N}{4}h'^4\,dz$$

$$F = \frac{1}{\sqrt{\phi(z_0)}}\int_{z_o}^{z_i} (Lgh^3 + M(hgh'^2 + h^2h'g') + Ng'h'^3)\,dz$$

$$C = \frac{1}{\sqrt{\phi(z_0)}}\int_{z_o}^{z_i} Lg^2h^2 + 2Mgg'hh' + Ng'^2h'^2\,dz$$

$$D = \frac{1}{2\sqrt{\phi(z_0)}}\int_{z_o}^{z_i} 2Lg^2h^2 + M(g^2h'^2 + g'^2h^2) + Ng'^2h'^2dz$$

$$E = \frac{1}{\sqrt{\phi(z_0)}}\int_{z_o}^{z_i} Lg^3h + M(hgg'^2 + g^2h'g') + Ng'^3h'\,dz$$

Magnetic Lens Aberration Integrals

$$K(z) = \frac{\eta^2B(z)^2}{8\hat{\phi}}$$

$$L(z) = \frac{\eta^4B(z)^4}{32\hat{\phi}^4} - \frac{\eta^2 2B(z)B''(z)}{8\hat{\phi}}$$

$$Q(z) = \frac{\eta B(z)}{4\sqrt{\hat{\phi}}}$$

$$P(z) = \frac{\eta^3B(z)^3}{16\hat{\phi}^{\frac{3}{2}}} - \frac{\eta B''(z)}{16\sqrt{\hat{\phi}}}$$

$$N = \frac{1}{2}$$

$$B = \int_{z_o}^{z_i} Lh^4 + 2Kh^2h'^2 + Nh'^4\,dz$$

$$F = \int_{z_o}^{z_i} Lgh^3 + Khh'(gh)' + Ng'h'^3\,dz$$

$$C = \int_{z_o}^{z_i} Lg^2h^2 + 2Kgg'hh' + Ng'^2h'^2 - K\,dz$$

$$D = \int_{z_o}^{z_i} Lg^2h^2 + K(g^2h'^2 + g'^2h^2) + Ng'^2h'^2 + 2K\,dz$$

$$E = \int_{z_o}^{z_i} Lg^3h + Kgg'(gh)' + Ng'^3h'\,dz$$

$$c = 2\int_{z_o}^{z_i} Pgh + Qg'h'\,dz$$

$$e = \int_{z_o}^{z_i} Pg^2 + Qg'^2\,dz\,dz$$

$$f = \int_{z_o}^{z_i} Ph^2 + Qh'^2\,dz$$