Analysis of Lorentz images of magnetic materials using a real space solution of the transport of intensity equation

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Keywords: phase retrieval, transport of intensity equation, finite difference methods

The transport of intensity equation (TIE) can be used to determine the phase shift ϕ of an electron wave from a series of defocused images of a specimen, according to the equation

$$\nabla(I\nabla\phi) = -\frac{2\pi}{\lambda}\frac{\partial I}{\partial z},$$

where I is the intensity, λ is the electron wavelength and z is the electron beam direction [1,2]. This solution for ϕ is unique if the phase shift at the boundary of the field of view is known. A fast and simple Fourier space approach can be used to find a solution to the TIE if I is taken to be constant [1] or to have a specific form [2]. Alternatively, $I\ \nabla\phi$ can be decomposed into the gradient of a scalar field and the curl of a vector field. The second term is then neglected and an approximate solution found for the phase if there are no first order zeros in the intensity [3]. Here, we illustrate a less restrictive approach provided by a real-space solution that accommodates intensity variations and allows the boundary phase to be specified arbitrarily. We approximate derivatives in the TIE by finite differences, and use a 'relaxation' approach to solve the differential equation. A refined phase estimate is then obtained iteratively using a finite difference method, given by the expression

$$\phi_{ij}^{n+1} = \frac{J_{ij} + (I_{i+1j} + I_{ij})\phi_{i+1j}^{n} + (I_{i-1j} + I_{ij})\phi_{i-1j}^{n} + (I_{ij+1} + I_{ij})\phi_{ij+1}^{n} + (I_{ij-1} + I_{ij})\phi_{ij-1}^{n}}{[4I_{ij} + I_{i+1j} + I_{i-1j} + I_{ij+1} + I_{ij-1}]}$$

where $J = [\, I(z + \Delta z) - I(z)\,]$ and the intensities are normalised to have unit mean. This approach can be applied to all of the pixels in an image successively and the process repeated until convergence is achieved. The result is then multiplied by an appropriate constant to provide the phase quantitatively. We have applied this approach to an energy-filtered defocus series of a 15-nm-thick Co film (Figure 1), which was recorded in magnetic-field-free conditions using a Philips CM300ST field emission gun transmission electron microscope equipped with a Lorentz lens. As the magnification varied by a factor of two across the series, the images were registered in magnification and orientation as well as position [4]. The phase shift recovered from two images on either side of focus is shown in Figure 2 for a boundary phase of zero. Figure 3 shows one component of the flux density inferred from Figure 2. Although such images appear to provide a reliable description of the induction in the specimen, the unknown boundary conditions result in the magnitude of the flux density varying across the field of view, as demonstrated by Figure 4 which shows contours of constant phase. Accordingly, we are exploring approaches for including constancy of the flux density in the solution to the TIE. We are also addressing the curious discrepancy that waves recovered from different focus levels appear to be inconsistent with each other, possibly as a result of contributions to the image intensity from a slowly-varying phonon background [5].

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5. The authors acknowledge financial support from the Royal Society and the EPSRC.

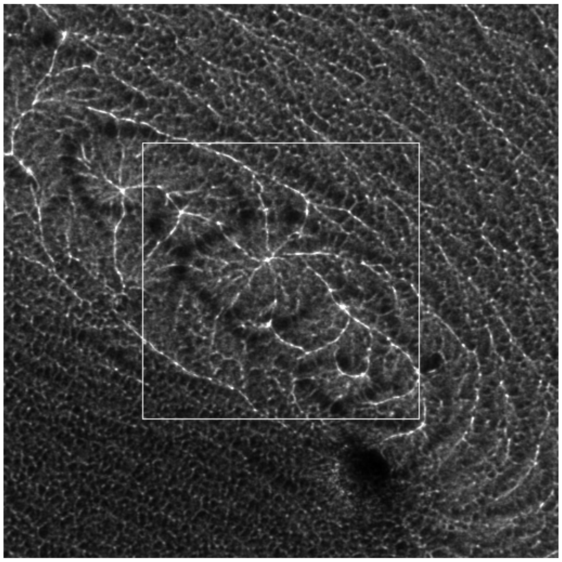

Figure 1. One member (slightly overfocus) of an energy-filtered defocus series of a polycrystalline Co film. The 2.6 μm square field marked is used in later figures.

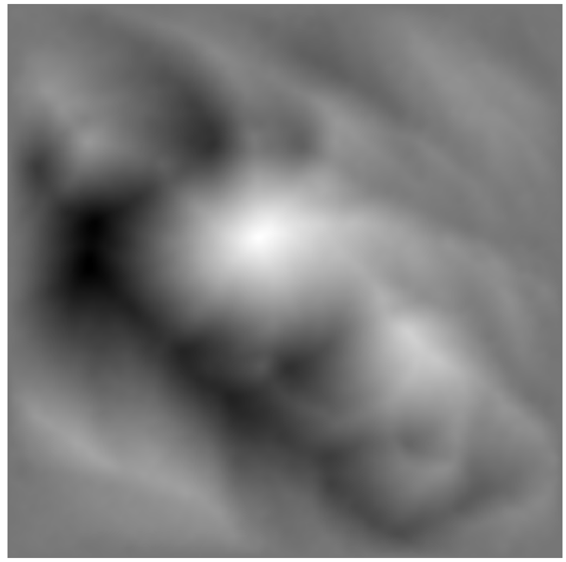

Figure 2. Phase shift recovered at Gaussian focus from the square region marked in Figure 1. The intensity range corresponds to a variation in phase of ~35 radians.

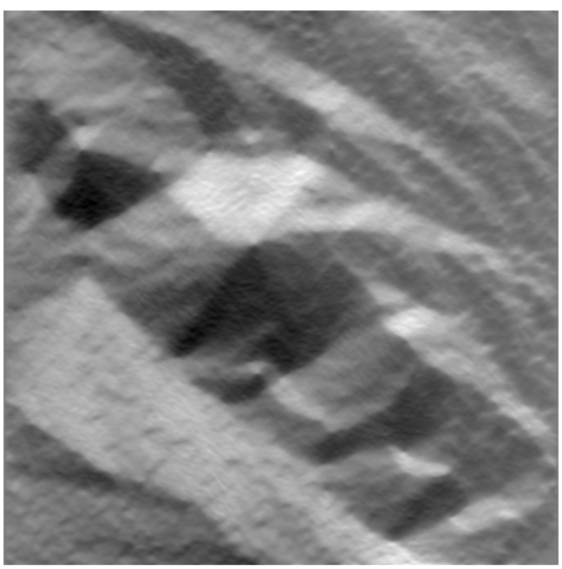

Figure 3. The *x*-component of the projected magnetic flux inferred from the phase shift in Figure 2. The maximum flux density is ~2T.

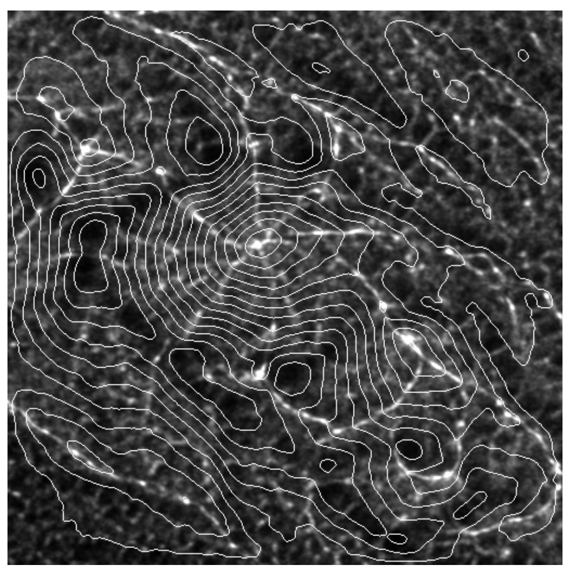

Figure 4. Contours of constant phase calculated from Figure 2 superposed on the square region marked in Figure 1.